Integrated Mathematics II

Adopted by the
Indiana State
Board of Education
September 2002

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. <u>What</u> they learn in mathematics and <u>how</u> they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Integrated Mathematics II:

Standard 1 — Algebra and Functions

Students draw and analyze graphs of linear inequalities in two variables and quadratics. They model data with linear equations and make predictions from the results.

Standard 2 — Geometry and Measurement

Students find lengths and midpoints of line segments, slopes, parallel and perpendicular lines, and equations of lines. They also construct lines and angles, explaining and justifying the processes they use. They define and construct altitudes, medians, bisectors, and triangles congruent to given triangles, as well as prove that triangles are congruent or similar and use properties of these triangles to solve problems involving lengths and areas. They find measures of sides, perimeters, and areas of triangles, justifying their methods. They define and understand the concepts of the trigonometric functions and know and use basic relationships among these functions. Students define and understand ideas related to circles (radius, tangent, chord, etc.).

Standard 3 — Data Analysis and Statistics

Students will interpret scatterplots and correlation coefficients and make predictions based on the least squares line.

Standard 4 — Probability

Students will construct probability distributions by simulation and explore the geometric distribution. They will use fundamental concepts of probability and counting principles to solve problems.

Standard 5 — Discrete Mathematics

Students will use graphs and networks to solve problems. They use matrices to organize data and solve problems.

Standard 6 — Trigonometry

Students will explore properties and applications of trigonometric ratios for right triangles.

Standard 7 — Mathematical Reasoning and Problem Solving

In a general sense, mathematics is problem solving. In all mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to justifying the steps in simplifying functions and solving equations and to deciding whether algebraic statements are true. They also learn how to use counterexamples to show that a general statement is false. Students apply these skills to making conjectures, using axioms and theorems, understanding the converse and contrapositive of a statement, constructing logical arguments, and writing geometric proofs. They also learn how to use counterexamples to show that a general statement is false.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, $\frac{1}{4}$, one divided by four, 0.25, $\frac{1}{8} + \frac{1}{8}$, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: +, =, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Standard 1

Algebra and Functions

Students graph linear inequalities in two variables and quadratics. They model data with linear equations.

IM2.1.1 **Graph a linear inequality in two variables.**

Example: Draw the graph of the inequality $6x + 8y \geq 24$ on a coordinate plane.

IM2.1.2 **Interpret given situations as functions in graphs, formulas, and words.**

Example: You and your parents are going to Boston and want to rent a car at Logan International Airport on a Monday morning and drop the car off in downtown Providence, R.I., on the following Wednesday. Find the rates from two national car companies and plot the costs on a graph. Decide which company offers the best deal. Explain your answer.

IM2.1.3 **Find a linear equation that models a data set using the median fit method and use the model to make predictions.**

Example: You light a candle and record its height in centimeters every minute. The results recorded as (time, height) are (0, 20), (1, 18.3), (2, 16.5), (3, 14.8), (4, 13.2), (5, 11.5), (6, 10.0), (7, 8.2), (9, 4.9), and (10, 3.1). Find the median fit line to express the candle's height as a function of the time and state the meaning of the slope in terms of the burning candle.

IM2.1.4 **Graph quadratic functions. Apply transformations to quadratic functions. Find and interpret the zeros and maximum or minimum value of quadratic functions.**

Example: Find the zeros for $y = x^2 - 4$. If $y = x^2 - 4$ has a maximum or minimum value, give the ordered pair corresponding to the maximum or minimum point.

Standard 2

Geometry and Measurement

Students identify and describe types of triangles. They define and apply the trigonometric relations. Students apply theorems to triangles and circles.

IM2.2.1 **Find the lengths and midpoints of line segments in one- or two-dimensional coordinate systems.**

Example: Find the length and midpoint of the line joining the points A (3, 8) and B (9, 0).

IM2.2.2 **Construct congruent segments and angles, angle bisectors, and parallel and perpendicular lines using a straight edge and compass, explaining and justifying the process used.**

Example: Construct the perpendicular bisector of a given line segment, justifying each step of the process.

IM2.2.3 **Find measures of interior and exterior angles of polygons, justifying the method used.**

Example: Calculate the measure of one interior angle of a regular octagon. Explain your method.

IM2.2.4 Identify and describe triangles that are right, acute, obtuse, scalene, isosceles, equilateral, and equiangular.

Example: Use a drawing program to create examples of right, acute, obtuse, scalene, isosceles, equilateral, and equiangular triangles. Identify and describe the attributes of each triangle.

IM2.2.5 Define, identify, and construct altitudes, medians, angle bisectors, and perpendicular bisectors.

Example: Draw several triangles. Construct their angle bisectors. What do you notice?

IM2.2.6 Use properties of congruent and similar triangles to solve problems involving lengths and areas.

Example: Of two similar triangles, the second has sides half the length of the first. The area of the first triangle is 20 cm^2. What is the area of the second?

IM2.2.7 Find and use measures of sides, perimeters, and areas of triangles. Relate these measures to each other using formulas.

Example: The gable end of a house is a triangle 20 feet long and 13 feet high. Find its area.

IM2.2.8 Prove, understand, and apply the inequality theorems: triangle inequality, inequality in one triangle, and the hinge theorem.

Example: Can you draw a triangle with sides of length 7 cm, 4 cm, and 15 cm?

IM2.2.9 State and apply the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle.

Example: In triangle ABC with right angle at C, draw the altitude $\overline{CD}$ from C to $\overline{AB}$. Name all similar triangles in the diagram. Use these similar triangles to prove the Pythagorean Theorem.

IM2.2.10 Use special right triangles (30° - 60° and 45° - 45°) to solve problems.

Example: An isosceles right triangle has one short side of 6 cm. Find the lengths of the other two sides.

IM2.2.11 Define and use the trigonometric functions (sine, cosine, tangent, cotangent, secant, cosecant) in terms of angles of right triangles.

Example: In triangle ABC, $\tan A = \frac{1}{5}$. Find $\sin A$ and $\cot A$.

IM2.2.12 Know and use the relationship $\sin^2 x + \cos^2 x = 1$.

Example: Show that, in a right triangle, $\sin^2 x + \cos^2 x = 1$ is an example of the Pythagorean Theorem.

IM2.2.13 Solve word problems involving right triangles.

Example: The force of gravity pulling an object down a hill is its weight multiplied by the sine of the angle of elevation of the hill. What is the force on a 3,000-pound car on a hill with a 1 in 5 grade? (A grade of 1 in 5 means that the hill rises one unit for every five horizontal units.)

IM2.2.14 Find the center of a given circle. Construct the circle that passes through three given points not on a line.

Example: Given a circle, find its center by drawing the perpendicular bisectors of two chords.

IM2.2.15 **Define and identify relationships among: radius, diameter, arc, measure of an arc, chord, secant, and tangent.**

Example: What is the angle between a tangent to a circle and the radius at the point where the tangent meets the circle?

IM2.2.16 **Prove theorems related to circles.**

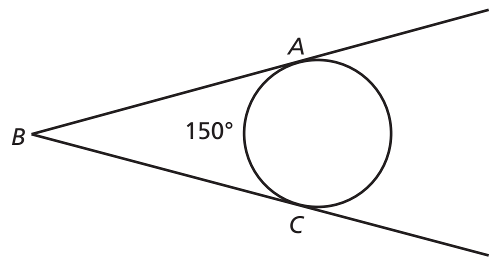
Example: Prove that an inscribed angle in a circle is half the measure of the central angle with the same arc.

IM2.2.17 **Construct tangents to circles and circumscribe and inscribe circles.**

Example: Draw an acute triangle and construct the circumscribed circle.

IM2.2.18 **Define, find, and use measures of arcs and related angles (central, inscribed, and intersections of secants and tangents).**

Example: Find the measure of angle *ABC* in the diagram below.



IM2.2.19 **Define and identify congruent and concentric circles.**

Example: Are circles with the same center always the same shape? Are they always the same size?

IM2.2.20 **Define, find, and use measures of circumference, arc length, and areas of circles and sectors. Use these measures to solve problems.**

Example: Which will give you more: three 6-inch pizzas or two 8-inch pizzas? Explain your answer.

IM2.2.21 **Describe sets of points on spheres: chords, tangents, and great circles.**

Example: On Earth, is the equator a great circle?

IM2

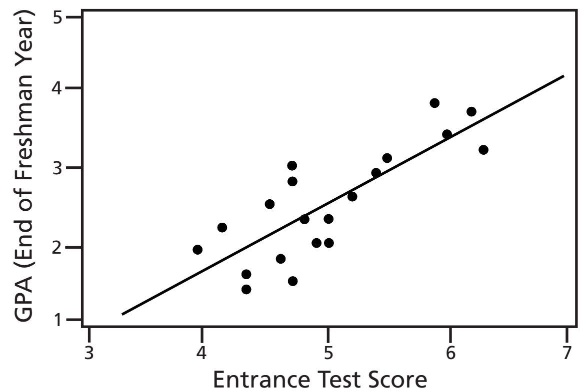
Standard 3

Data Analysis and Statistics

Students interpret scatterplots and analyze correlation.

IM2.3.1 Describe the association between two variables by interpreting a scatterplot.

Example:



The director of admissions of a small college administered a newly designed entrance test to 20 students selected at random from the new freshman class in a study to determine whether a student's grade point average at the end of the freshman year (y) can be predicted from the entrance test score (x). The scatterplot shows a positive relationship between entrance test score and GPA at the end of the freshman year. As the entrance test score increases, the student's GPA at the end of the freshman year also increases.

IM2.3.2 Interpret correlation coefficients.

Example: In the example in indicator 3.1, the correlation coefficient for the student's GPA at the end of the freshman year and the entrance test score is .809, resulting in a strong linear relationship between the two variables.

IM2.3.3 Make predictions from the least squares regression line or its equation.

Example: In the example in indicator 3.1, based on the regression line and the regression equation $y = -1.70 + 0.840x$, a student who scores a 6 on the entrance test score could expect to receive a GPA score of 3.34 at the end of the freshman year.

IM2.3.4 Understand that a correlation between two variables does not necessarily imply one directly causes the other.

Example: Comparing weekly flu medication sales and weekly sweater sales for an area with extreme seasons would exhibit a positive association given that sales for both tend to increase during the winter and decrease during the summer. However, the question remains, does one cause the other to occur?

IM2.3.5 Understand the effects of outliers on correlation coefficients, on the least squares regression line, and on the interpretations of correlation coefficients and regression lines in real-life contexts.

Example: In the example in indicator 3.1, consider the effect of adding a student who scores a 3 on the entrance test but received a 4 for the GPA at the end of the freshman year. This outlier would significantly lower the correlation coefficient to 0.386 and change the regression line to $y = 0.75 + 0.371x$. No longer does this data show a significant linear relationship, which, if used, would lead to wrong conclusions for the director of admissions.

IM2

Standard 4

Probability

Students construct probability distributions, understand fundamental probability concepts, and use counting principles.

IM2.4.1 **Construct a probability distribution by simulation and use it to understand and analyze the probabilistic situation.**

Example: Jamie is a basketball player who makes 70 percent of her free throws over the course of a season. In a key game, Jamie shoots 10 free throws. What is the probability she makes five of her free throws? Simulate 10 free throws shot independently using random numbers from 1 to 10 with 1 to 7 representing a success and record 100 trials.

IM2.4.2 **Explore the geometric, or waiting-time, distribution.**

Example: A card is drawn from a deck of cards. Observe the card and replace the card within the deck. What is the probability you will observe a jack on the sixth attempt?

$P(X = 6) = (12/13)^5\ (1/13) = .0516$

IM2.4.3 **Understand fundamental concepts of probability (e.g., independent events, multiplication rule, and expected value).**

Example: Decide if the following events are independent: Bob draws a card from a standard deck of cards, replaces it, and then draws a second card. Explain your answer.

IM2.4.4 **Understand and apply counting principles to compute combinations and permutations.**

Example: There are 5 students who work in a bookshop. If the bookshop needs 3 people to operate, how many days straight could you work without the same group of students working twice?

IM2.4.5 **Use the basic counting principle, combinations, and permutations to compute probabilities.**

Example: You are on a chess team made up of 15 players. What is the probability that you will be chosen if a 3-person team is selected at random?

Standard 5

Discrete Mathematics

Students use graphs and networks as mathematical models and use matrices to solve problems.

IM2.5.1 **Experience in mathematical modeling by building and using vertex-edge graph models to solve problems in a variety of real-world settings.**

Example: Five students are to play in a ping-pong tournament. Each student is to play all the other players once. Draw a vertex-edge graph to show the winners of each match by drawing an arrow from the winner to the defeated student.

IM2.5.2 **Develop the skill of algorithmic problem solving: designing, using, and analyzing systematic procedures for problem solving.**

Example: Sarah wants to fly her aircraft from Maui to a remote island off its shore. To determine the amount of gas needed for a round trip, she must know how far the island is from Maui. Given the coordinates of the island and Maui, how many kilometers will she travel round trip?

IM2.5.3 Optimize networks in different ways and in different contexts by finding minimal spanning trees, shortest paths, and Hamiltonian paths.

Example: A family is planning a trip to see the Grand Canyon, Wisconsin Dells, Yellowstone National Park, Pikes Peak, Little Big Horn, and Mount Rushmore. The family wants to determine a path with the least amount of time on the road. What would be the best path to take?

IM2.5.4 Use matrices to organize and display data in a variety of real-world settings.

Example: Develop a matrix for the ping-pong tournament in the example in indicator 5.1 to rank each of the five students with the results from their win/loss records.

IM2.5.5 Develop mathematical modeling skills by building matrix models and then apply the models to solve problems.

Example: To prepare for a dance, a school needs to rent 100 chairs, four large tables, and 10 punch bowls. Rental prices were collected from two rental shops with the following matrix representing the two rental shops:

$$\begin{array}{l c} & \begin{array}{cc} R_1 & \quad R_2 \end{array} \\ \begin{array}{l} \text{Chairs} \\ \text{Tables} \\ \text{Bowls} \end{array} & \begin{bmatrix} \$2 & \$2.50 \\ \$20 & \$15 \\ \$6 & \$4 \end{bmatrix} \end{array}$$

Which rental shop, R_1 or R_2, has the lowest price for the group of items?

IM2.5.6 Apply matrix operations to solve problems (i.e., row sums, scalar multiplication, addition, subtraction, and matrix multiplication).

Example: Use matrix multiplication to solve the problem in indicator 5.5.

Solve $[100 \quad 4 \quad 10] \bullet \begin{bmatrix} 2 & 2.5 \\ 20 & 15 \\ 6 & 4 \end{bmatrix} = [340 \quad 350]$.

IM2.5.7 Use matrices and inverse matrices to answer questions that involve systems of linear equations.

Example: Solve the system of equations using matrices:
$2x + 20y + 6z = 340$
$2.5x + 15y + 4z = 350$

IM2.5.8 Build and use matrix representations to model polygons, transformations, and computer animations.

Example: Transform the following matrix, which represents the points on a triangle, to reflect across the y-axis to resemble a spinning effect.

Triangle $ABC = \begin{bmatrix} -2 & 2 & 0 \\ -1 & 1 & 5 \end{bmatrix}$

Standard 6

Trigonometry

Students apply trigonometric ratios to right triangles.

IM2.6.1 Explore properties and applications of the sine, cosine, and tangent ratios for the lengths of sides of right triangles.

Example: A farmer needs to change a bulb that is 35 feet high on the side of his barn. Because of an automatic water dispenser at the base of the barn, directly under the light, the angle at which the ladder will be placed on the ground to the barn is a maximum of 70°. The farmer only has a 30-foot ladder. Assuming the farmer is six feet tall, is the ladder tall enough for the farmer to change the light bulb?

Standard 7

Mathematical Reasoning and Problem Solving

Students use a variety of strategies to solve problems and develop and evaluate mathematical arguments and proofs.

IM2.7.1 Use the properties of the real number system and the order of operations to justify the steps of simplifying functions and solving equations.

Example: Solve $3x + 5 = 2x - 1$, explaining why you can take each step.

IM2.7.2 Make conjectures about geometric ideas. Distinguish between information that supports a conjecture and the proof of a conjecture.

Example: Calculate the ratios of side lengths in several different-sized triangles with angles of 90°, 50°, and 40°. What do you notice about the ratios? How might you prove that your observation is true (or show that it is false)?

IM2.7.3 Write and interpret statements of the form "if – then" and "if and only if."

Example: Decide whether this statement is true: "If today is Sunday, then we have school tomorrow."

IM2.7.4 State, use, and examine the validity of the converse, inverse, and contrapositive of "if – then" statements.

Example: In the example in indicator 7.3, write the converse of the statement.

IM2.7.5 Write geometric proofs, including proofs by contradiction and proofs involving coordinate geometry. Use and compare a variety of ways to present deductive proofs, such as flow charts, paragraphs, and two-column and indirect proofs.

Example: In triangle LMN, $LM = LN$. Prove that $\angle LMN \cong \angle LNM$.

IM2.7.6 Perform basic constructions, describing and justifying the procedures used. Distinguish between constructing and drawing geometric figures.

Example: Construct a line parallel to a given line through a given point not on the line, explaining and justifying each step.

IM2.7.7 Decide if a given algebraic statement is true always, sometimes, or never (statements involving quadratic expressions).

Example: Is the statement $x^2 - 4 \geq 0$ true for all x, for some x, or for no x? Explain.

IM2.7.8 Understand that the logic of equation solving begins with the assumption that the variable is a number that satisfies the equation and that the steps taken when solving equations create new equations that have, in most cases, the same solution as the original. Understand that similar logic applies to solving systems of equations simultaneously.

Example: Try "solving" the equation $4x - 9 = 5 - 2(8 - 2x)$ and explain what went wrong.

IM2.7.9 Use counterexamples to show that statements are false.

Example: Show by an example that this statement is false: two triangles with the same area and same perimeter are congruent.